

Evolution of the universe driven by a mass dimension one fermion field

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This paper study the evolution of the universe filled by a neutral mass dimension one fermionic field, sometimes called Elko. The numerical analysis of the coupled system of equations furnish a scale factor evolution that exactly reproduces the inflationary phase of the universe followed by a dark matter evolution. The time scale shows that nearly an exponential inflation occurs up to about 10^{-32} s and then the evolution follows as $t^{2/3}$ up today. The analysis was performed with a potential containing a quadratic mass term and a quartic self-interaction term, which follows naturally from the theory of mass dimension one fermions, and it was verified that the inflationary phase can be driven just by the self-interaction term. The number of e-foldings of inflation was found to be strongly dependent on the initial condition of the Elko field, as occurs in chaotic inflationary models. The growth of the scale factor can be extrapolated up to present time. A nowadays accelerated expansion can also be implemented by adding a constant term into the potential, exactly as a cosmological constant term.

I. INTRODUCTION

The search for a model correctly describing all the evolution of the universe is an old problem in cosmology. In the current model, the universe starts in a very hot and dense phase known as the big bang¹ driven by quantum effects based on models as supersymmetry, supergravity, extra dimensions, superstrings, among others. The quantum effects are dominant while the energy density is greater than Planck energy density m_{pl}^4 , characterized by the Planck mass $m_{pl} \simeq 1.22 \times 10^{19}$ GeV. The age of the universe is about 10^{-43} s at this stage. From 10^{-43} s to about 10^{-34} s the universe is still expanding and cooling in the so called pre-inflationary phase and its temperature is about 10^{14} GeV. From 10^{-34} s to 10^{-32} s the universe undergoes the so called inflation, where the scale factor $a(t)$ growth for a factor of about 10^{43} . Such phase is necessary in order to solve some problems as the flatness problem, the monopole and relics problems, the horizon problem and homogeneity [2, 3]. After such very abrupt expansion, nearly exponential, the universe passes a reheating phase, where the ordinary matter (radiation first and then baryonic matter) start

to dominate and several process occur, as the end of electroweak unification, the quark-hadrons transition, the nucleosynthesis of light elements and the formation of structures, as galaxies and cluster of galaxies. The scale factor growth to about 10^{74} up today, with an age of about 10^{18} s. Finally, very recently the universe starts a new accelerating phase, dominated by a cosmological constant term or a dark energy fluid. This is a very brief history of the universe.

An unified model that could describes all the phases of evolution of the universe is a difficult task. The several orders of magnitude involved from the inflation to recent cosmic acceleration forced the researches to divide the evolution of the universe in different parts, each one characterized by different kind of particles that dominate at different stages. These ingredients form the standard model of cosmology. The inflationary phase of the universe can be constructed with a single scalar field that drive the inflation while the scalar field rolls down to the bottom of its potential. Several potentials that satisfies the fine tunings of the inflationary phase have been studied in last decades (see [2, 3] for a review and [4] for observational constraints on several potentials.). After inflation, the scalar field ends in a rapid oscillation around the minimum of its potential and its energy is transferred to the baryonic particles in the next phase of evolution, including radiation, in a process known as reheating. Radiation and baryons corresponds to about 5% of the total content

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¹ See pages 73 and 274 of [1] for this brief resume of the thermal history of the universe.

of the universe. The scalar field do not act anymore and the universe goes to the radiation dominated phase and subsequently to the matter dominated phase. During the matter dominated phase there is the necessity to add a new kind of non-baryonic matter in the universe in order to correctly explain structure formation, thus the standard model admits the presence of the dark matter, about 25% of the total content of the universe. After evolving dominated by dark matter, observations shown that a recent accelerating phase of expansion needs a new ingredient, the so called cosmological constant or even a new kind of energy, called dark energy, acting as a vacuum energy density and representing about 70% of the total energetic content.

As previously stated, a model describing all phases of the evolution of the universe is a challenge for the present cosmology. Recently, a new class of fermions with mass dimension one named Elko [5–10] was proposed and established in solid quantum bases very recently [10, 11] (see also [12]). Such new spinorial field is a natural candidate to dark matter in the universe since that it is constructed as spin-1/2 particles describing fermions that are eigenstate of the charge conjugation operator, being neutral and weakly coupled to the electromagnetic sector of the standard model of particles. Several cosmological applications of such field have recently appeared in the literature [13–33]. In [33] has been studied numerically the possibility of the fermionic Elko field driven the evolution of the universe from inflation, passing by the dark matter dominated epoch and finishing with the recent cosmic acceleration. The potential used was of symmetry breaking type and the mass of the Elko field needed to drive the evolution was found to be about 10^9 GeV , a very huge value. Due to numerical difficulties, the time scale used was very short, not allowing a complete estimate of the growth of the scale factor up today.

In the present paper the numerical solution concerning the Elko field evolution in the presence of a potential with a quadratic mass term and quartic self interaction term was obtained. Contrary to the previous paper, here it is shown that all the evolution can be driven by the quartic self interaction term, leaving the mass term free to be smaller if necessary, depending on other tests to constrain it. Also, the time scale used here allows to estimate the real growth of the scale factor up today.

II. DYNAMIC EQUATIONS FOR ELKO FIELD

In the flat space-time, there are four Elko spinors satisfying invariance by the charge conjugation operator C , they are labelled as $\lambda_\beta^{S/A}$ where S stands for Self-conjugate and A for Anti-self-conjugate. The index β stands for two possible helicities. They satisfies $C\lambda_\beta^{S/A} = \pm\lambda_\beta^{S/A}$ [5–12] and are normalized satisfying the relation $\bar{\lambda}_\beta(\mathbf{k})^{S/A}\lambda_{\beta'}(\mathbf{k})^{S/A} = \pm 2m\delta_{\beta\beta'}$, where $\lambda_{\beta'}(\mathbf{k})^{S/A}$ and $\bar{\lambda}_\beta(\mathbf{k})^{S/A}$ are the usual spinor and its dual, respectively. The dual have been redefined recently [10–12] in order to maintain locality and Lorentz covariance. The corresponding quantum fields can be redefined in order to satisfies a normalization relation as $\bar{\lambda}\lambda = \pm 1$, where λ stands here for any of the four spinors, with normalization $+1$ for the two self-conjugate and -1 for the two anti-self-conjugate.

In this paper it was used just one fermionic Elko field satisfying a positive normalization. Also, in order to use the Elko field in a curved background, it was factored out the time dependence of the field as [18–33] $\Lambda = \phi(t)\lambda$ and the action for the model reads:

$$S = \int d^4x \sqrt{-g} \left[-\frac{\tilde{R}}{2\kappa^2} + \frac{1}{2}g^{\mu\nu}\tilde{\nabla}_\mu \bar{\Lambda} \tilde{\nabla}_\nu \Lambda - V(\bar{\Lambda} \Lambda) \right], \quad (1)$$

where $\kappa^2 \equiv 8\pi G$ with $c = 1$. The tilde denotes the presence of torsion terms into the Ricci scalar \tilde{R} and covariant derivatives, namely, $\tilde{\nabla}_\mu \Lambda \equiv \partial_\mu \Lambda - \Gamma_\mu \Lambda$ and $\tilde{\nabla}_\mu \bar{\Lambda} \equiv \partial_\mu \bar{\Lambda} + \bar{\Lambda} \Gamma_\mu$, where Γ_μ is the connection associated to spinor fields, containing the spin connections. For the potential it was used a constant term plus a quadratic mass term and a quartic self-interacting term, namely:

$$V = v_0 + \frac{1}{2}m^2 \bar{\Lambda} \Lambda + \frac{\alpha}{4}(\bar{\Lambda} \Lambda)^2 = v_0 + \frac{1}{2}m^2\phi^2 + \frac{\alpha}{4}\phi^4, \quad (2)$$

where v_0 is a constant, m is the physical mass of the field and α a dimensionless coupling. Such kind of potential follows naturally from the theory of mass dimension one fermions [6].

In a flat Friedmann-Robertson-Walker (FRW) metric $ds^2 = dt^2 - a(t)^2[dx^2 + dy^2 + dz^2]$, the two Friedmann equations and the dynamic field equation for $\phi(t)$ for such action can be obtained [27, 28]:

$$H^2 = \frac{\kappa^2}{3} \left(1 + \frac{\kappa^2\phi^2}{8} \right) \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right], \quad (3)$$

$$\dot{H} = -\frac{\kappa^2}{2} \left(1 + \frac{\kappa^2 \phi^2}{8} \right) \left[\dot{\phi}^2 - \frac{1}{2} \frac{H \phi \dot{\phi}}{(1 + \kappa^2 \phi^2/8)^2} \right], \quad (4)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} - \frac{3}{4} \frac{H^2 \phi}{(1 + \kappa^2 \phi^2/8)^2} = 0, \quad (5)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and a dot stands for time derivative. Just two of that equations are independent, since that taking the time derivative of the first and using the second the third is obtained. Such equations generalizes that one for a standard scalar field. The second term inside curl bracket of (3)-(4) and the last terms of (4)-(5) are not present in the standard scalar field equations. This shows that the Elko dynamic field equations are much richer than the scalar field ones. Such additional terms comes from the fact that the fermionic Elko field must be coupled to torsion in an Einstein-Cartan framework and also due to spin connections terms. In particular, since that $\kappa^2 = 8\pi G = 8\pi/m_{pl}^2$, in the limit $\phi \ll m_{pl}$ and $H\phi \ll \dot{\phi}$ these additional terms can be discard and the equations are exactly like that ones for the standard scalar field. Thus the limit $\phi \sim m_{pl}$ represents a kind of transition of the Elko field from a high energy regime dominated by torsion and spin connection terms when $\phi \gtrsim m_{pl}$ and a low energy regime of a standard scalar field free of torsion and spin connection terms when $\phi \ll m_{pl}$. Surprisingly, it has been found numerically that such transition from a high energy regime to a low energy one can be the responsible for the inflationary phase of the universe.

Given a potential $V(\phi)$, the slow-roll parameters [2] ϵ and η for the Elko system were obtained in [33]:

$$\epsilon(\phi) \equiv \frac{|\dot{H}|}{H^2} \simeq \frac{1}{\kappa^2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2, \quad (6)$$

$$|\eta(\phi)| \equiv \left| \frac{\ddot{\phi}}{H\dot{\phi}} \right| \simeq \left| \frac{1}{\kappa^2} \frac{V''(\phi)}{(1 + \kappa^2 \phi^2/8)V(\phi)} \right|, \quad (7)$$

and the condition to inflation occurs is $\epsilon(\phi) \ll 1$ and $|\eta(\phi)| \ll 1$. In which follows it will set the initial conditions and values of the parameters in order to make a numerical analysis of the system.

III. NUMERICAL RESULTS

In order to perform a numerical analysis² for the evolution of the scale factor $a(t)$ and $\phi(t)$ it was chosen to work with the coupled equations (3) and (5). It is also necessary to fix some parameters of the potential (2) and initial conditions satisfying the slow-roll conditions (6)-(7). Since that (3) is a first order differential equation in $a(t)$ it needs just one initial condition at t_i , and it was chosen $a(t_i) = 1$. Equation (5) is a second order differential equation and needs two initial conditions. In order to test the hypothesis that inflation occurs due to a transition of the Elko field from a high energy scale $\phi \gtrsim m_{pl}$ to a low energy regime $\phi \ll m_{pl}$, it was chosen $\phi(t_i) \equiv \phi_i = 1.75m_{pl} = 3.25 \times 10^{43}\text{s}^{-1}$, similar to chaotic inflationary model by Linde [34]. The inflation must occur when $\phi(t)$ decay from ϕ_i to zero, at the bottom of the potential. It has been also considered that before inflation occurs the universe was filled with a nearly homogeneous and isotropic gas of Elko particles, nearly at rest, so it was chosen $\dot{\phi}(t_i) \equiv \dot{\phi}_i = 0$. Thus the unique energy present at the beginning is its potential energy $V(\phi_i)$.

After some numerical analysis it was found that the beginning of the evolution, namely the phase that includes the inflation, can be driving just by the self-interacting quartic term of the potential if $m^2 \ll \frac{1}{2}\alpha\phi_i^2$, which is obvious since that it is proportional to ϕ^4 while the mass term is proportional to ϕ^2 and the initial condition for ϕ is nearly greater than the Planck mass. By choosing $\alpha = 4 \times 10^{-20}$ the condition for the mass is $m \ll 2.5 \times 10^{-10}m_{pl} \simeq 3 \times 10^9\text{GeV}$. It was chosen $m = 1.0\text{GeV}$ so that it does not contribute to the evolution at the beginning. When the field decays below about $10^{-10}m_{pl} \simeq 10^{33}\text{s}^{-1}$ the quadratic mass term starts to dominate. Numerically this occurs at about 10^{-5}s , very far in the past. Finally, the value for the constant v_0 of the potential was chosen as $v_0 = 0$ in the first analysis. Having stated the initial conditions and the value of the parameters, the slow-roll parameters can be tested in order to warranty the inflationary phase to occur. The values are $\epsilon \simeq 0.21$ and $\eta \simeq 0.015$, which are reasonable values lesser than 1. It is also important to satisfies the con-

² It was used the Maple 15 Software, whose numerical solutions are found by a Fehlberg fourth-fifth order Runge-Kutta method with degree four of interpolation.

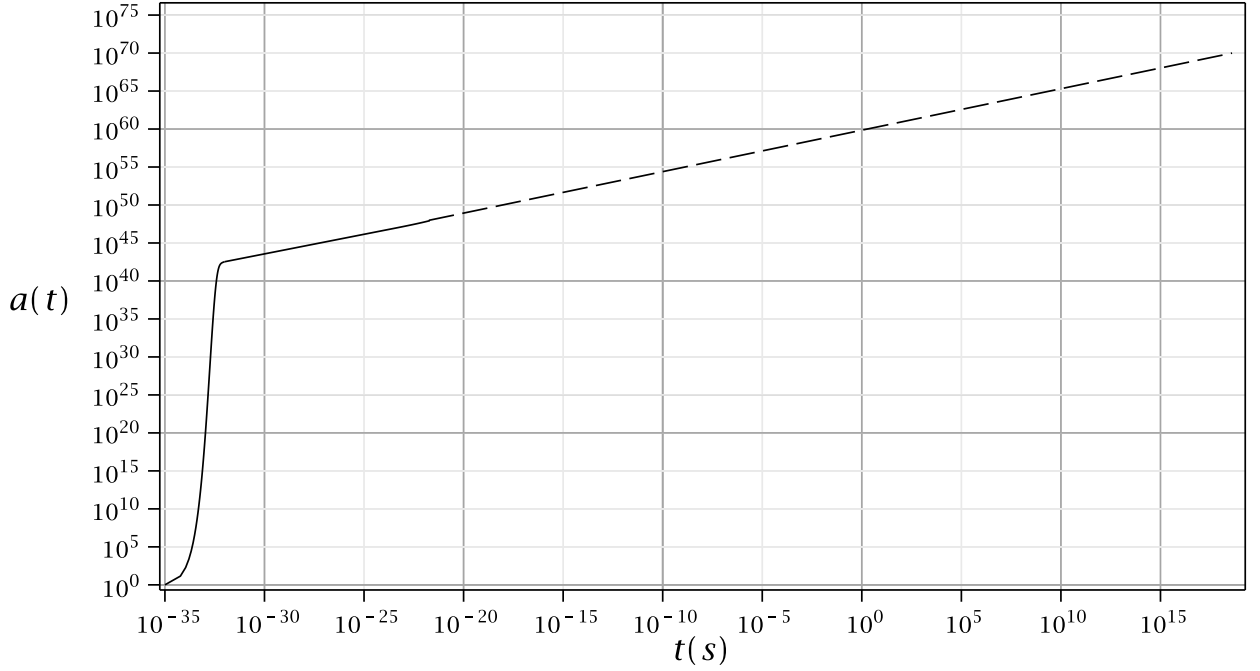


FIG. 1: Numerical result for the evolution of the scale factor $a(t)$ with time. Up to about $t \simeq 10^{-20}$ s (solid line) the figure was obtained numerically, showing that after a inflationary phase the evolution follows a power law as $t^{2/3}$. After $t \simeq 10^{-20}$ s (dashed line) the straight line was extrapolated, showing the growth of the scale factor up today, about 10^{18} s.

dition $V(\phi_i) \ll m_{pl}^4$ to warranty that the total energy of the field is below the Planck scale, so that quantum effects are not present. Such condition is satisfied due to tiny value of α . Now it is possible to make a numerical analysis of the system of equations (3) and (5). In order to have the time scale in seconds, it was used $\kappa = \sqrt{8\pi}/m_{pl} = 4.1 \times 10^{-19} \text{GeV}^{-1} = 2.7 \times 10^{-43} \text{s}$. The initial time t_i was chosen as $t_i = 10^{-35} \text{s}$. It is expected that inflation occurs up to about 10^{-32}s and after that the universe expand in a nearly power law in time.

Figure 1 contains the main result of the paper. The evolution of the scale factor $a(t)$ with time (in seconds) for the above initial conditions and parameters of the potential shows that inflation in fact occurs up to about 10^{-32}s and the scale factor growth for about 10^{43} order of magnitude, which corresponds to about 99 e-foldings. After that the universe enters a power law evolution, which is indicated by the straight line in the logarithm scales. In a non-logarithm scales it was verified that the evolution is of the type $a(t) \propto t^{2/3}$. Such behaviour was also observed in [33] for the symmetry breaking potential. The reason is obvious. As the field rolls down to the bottom of the potential its values goes to zero, and the pressure of the field also

goes to zero. So the Elko field satisfies a dust equation of state type, exactly as desired for a dark matter fluid. Thus the Elko field act as a dark matter in the rest of the evolution of the universe for a long time. Having nearly null kinetic energy the Elko particles could be attracted to other local potentials, initiating the growth of small structures, as dark matter halos. Remember that it is one the main characteristic of the Elko field, a candidate to dark matter particle. The numerical analysis of the equation of state parameter $\omega \equiv p(\phi)/\rho(\phi)$ also indicates that ω starts from -1 then begins to oscillate around $\omega = 0$ for the rest of its evolution. Another very interesting and surprising results from the Figure 1 is that today, when $t \simeq 10^{18} \text{s}$, the scale factor growth to about 10^{70} , in a good accord to the previous estimate from [1] of about 10^{74} for the standard model. The numerical analysis was done and is represented by the solid line, while the continuation of the straight line was extrapolated due to a very long time of computations after about 10^{-20}s . Nevertheless, it is obvious that such extrapolation can be done, since that the model has no other kind of particle present to dominate in other phases.

Figure 2 (a) shows the numerical results for the decaying of the Elko field $\phi(t)$ from its initial value

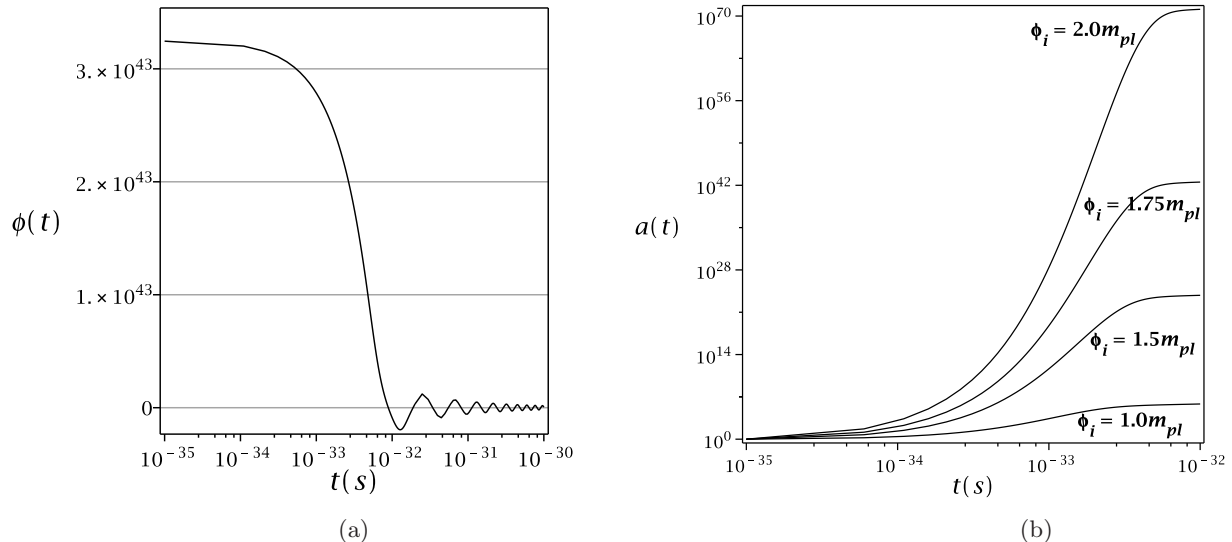


FIG. 2: (a) - Evolution of the Elko field $\phi(t)$ to its minimal value and its oscillation with time before to slow-roll to the bottom of the potential. The units of vertical scale is $[s^{-1}]$. (b) - Detailed view of the inflationary phase and its strong dependence on the initial value ϕ_i .

$\phi_i = 1.75m_{pl}$. Before reach the bottom of the potential the field oscillates for a long time around its minimal value. Figure 2 (b) shows in detail just the inflationary phase, showing a strong dependence with the initial value ϕ_i of the field. The other parameters are the same of previous analysis. It is possible to see clearly that the number of e-foldings of the inflation is strongly dependent on the initial condition for the field. This is a kind of fine tuning for the model.

IV. CONCLUDING REMARKS

The numerical analysis of the coupled system of equations concerning a homogeneous and isotropic distribution of Elko fields filling the whole universe was performed. The potential under which the Elko field slows down is a quadratic mass term and a quartic self-interaction on the field. It was found that the evolution for the scale factor reproduces the expected exponential growth during the inflationary epoch up to about 10^{-32} s driven just by the self-interaction term of the potential, and after inflation the universe evolves dominated by a dark matter fluid of Elko particles with nearly zero kinetic and potential energy. This opens the possibility to the Elko field be attracted to other local potentials and start to form the first structures of the universe, contributing to dark matter haloes for instance. Notice from Figure 1 and Figure 2 (a) that inflation occurs exactly when the field goes from ϕ_i to

zero. As the field decays to the bottom of its potential it starts to oscillate and its amplitude diminishes with time.

The growth of the scale factor up today can be extrapolated from the numerical analysis into Figure 1 and it was found that today the scale factor is about 10^{70} order of magnitude greater than its initial value. It is in good accord to an estimate of about 10^{74} for the standard model of cosmology. Certainly the difference is due to the fact that here it has been considered the Elko as the only particle, not including radiation and baryonic matter. A more complete model must also include such particles as well.

In this analysis the mass of the Elko field was taken as 1.0GeV and its effects during the inflationary phase is not important, thus the mass of the Elko field must be estimated by others observational constraints, as when starting to form structures for instance. It was also obtained (Figure 2 (b)) that the number of e-foldings during the inflationary phase is strongly dependent on the initial value of the field, here taken as $\phi_i \gtrsim m_{pl}$, as occurs in chaotic inflationary models.

In order to also address the problem of recent cosmic acceleration, the constant term of potential, namely v_0 , must be taking as non null, exactly as a cosmological constant term. Thus, all phases of evolution of the universe can be obtained in a very elegant way by the dynamic evolution of the mass dimension one Elko field in a FRW background.

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